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Decentralized Robust Control for Interconnected Nonlinear Systems

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Decentralized robust control of strongly interconnected uncertain systems is addressed in this paper. The uncertainties we consider here may include parameter uncertainty and input disturbance that may be nonlinear and (possibly fast) time-varying. We show specially how both internal and external uncertainties are taken into considerations. This work covers a broad class of large-scale systems since the current consideration renders all previous settings as special cases. Stability analysis with the proposed controllers is provided.

Key Words: Decentralized Control, Robust Control, Large-Scale Systems, Interconnected Uncertain Nonlinear Systems

1. Introduction

A large-scale system can be considered as a set of interconnected subsystems, which is often encountered in practical applications, for example, power networks and transportation systems. The important trend in control design for these large-scale systems is decentralized control where the control law of each subsystem is based only on its own information. The advantage of this aspect in controller design is to reduce complexity and a formidable amount of information transmission and therefore allows the control implementation to be more feasible (Siljak 1991). The research on decentralized control has been prolific. Important and representative work on decentralized control of large-scale uncertain systems can be found in Gavel and Siljak (1989), Ikeda and Siljak (1990), Ohta et al. (1986), Sezer and Siljak (1981), Siljak (1989), and their bibliographies.

For an uncertain large-scale system, it usually turns out that each subsystem may possess internal uncertainty. Besides, there may be uncertainties in the interconnections. The uncertainties we consider here may include parameter uncertainty and input disturbance that may be nonlinear and (possibly fast) time-varying - their realizations are often unknown. It may be also difficult to acquire their statistical properties a priori. Therefore a potential approach for handling such uncertainty is to utilize the knowledge related to its possible bound. The controller design should then be based on only this knowledge. This is the spirit of the deterministic approach to uncertain large-scale systems, which we shall adopt in this paper. A survey of related work using this approach for centralized control design can be found in Corless and Leitmann (1988).

The past work on decentralized robust control has been limited to the considerations of nonlinear systems where only weak interconnections arise (Chen, 1988). Once strong interconnections arise, the above passive analysis may be sometimes overconservative in terms of providing a quantitative measure of the threshold. Gavel and Siljak (1989) was among the first to study such an issue. Adaptive control is constructed, which is able to suppress the interconnection if its bound is proportional to the state norm. Later Chen et al.

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(1991) proposed a non-adaptive control design for a setting with cone-bounded interconnection, which renders the previous bound into a special case.

The present work extends the setting of Chen et al. (1991) to arbitrary bound. Loosely speaking, the previous work considers linear bound and the current work considers nonlinear bound.

2. Interconnected Systems

Consider a class of uncertain large-scale systems that are composed of N interconnected subsystems S_i described by (throughout this paper, for the sake of brevity, arguments are sometimes omitted when no confusion is likely to arise)

$$S_i:\ \dot{x}_i(t)=f_i(x_i(t),t)+\Delta f_i(x_i(t),\sigma_i(t),t)+[B_i(x_i(t),t)+\Delta B_i(x_i(t),\sigma_i(t),t)]\cdot u_i(t)+g_i(x(t),\sigma_i(t),t)$$

$$x_i(t_0)=x_{i0} \tag{1}$$

$$x(t)=[x_1^T(t),x_2^T(t),\cdots,x_N^T(t)]^T\in R^n,\quad n=\sum_{i=1}^{N}n_i \tag{2}$$

where $i\in N$, $N=\{1,2,\cdots,N\}$. Here $t\in R$ is the independent variables, $x_i(t)\in R^{n_i}$ is the state, and $u_i(t)\in R^{m_i}$ is the control. Uncertainty in the system description is modelled by an unknown Lebesgue measurable function $\sigma_i(\cdot): R\to\Sigma_i$, where the uncertainty bounding set $\Sigma_i\subset R^{l_i}$ is a known compact set. The known functions $f_i(\cdot)$: $R^{n_i}\times R\to R^{n_i}$ and $B_i(\cdot)$: $R^{n_i}\times R\to R^{n_i\times m_i}$ and the (known or unknown) functions $\Delta f_i(\cdot)$: $R^{n_i}\times\Sigma_i\times R\to R^{n_i}$, $\Delta B_i(\cdot)$: $R^{n_i}\times\Sigma_i\times R\to R^{n_i\times m_i}$ and $g_i(\cdot)$: $R^{n_i}\times\Sigma_i\times R\to R^{n_i}$ are Lebesgue measurable in t and continuous in other arguments.

Regarding the uncertain system (1), some structural conditions are imposed on the uncertainty. Standard notation is employed. If the eigenvalues of a matrix Ξ are real, $\lambda_m[\Xi]$ and $\lambda_M[\Xi]$ (or $\lambda_m(\Xi)$ and $\lambda_M(\Xi)$) denote the minimum and maximum eigenvalues of Ξ. Vector norms are Euclidean and matrix norms are the corresponding induced ones; thus $\|\Xi\|=[\lambda_M(\Xi^T\Xi)]^{1/2}$.

Assumption 1. Maching of Uncertainties. There exist continuous functions $d_i(\cdot)$: $R^{n_i}\times\Sigma_i\times R\to R^{m_i}$, $E_i(\cdot)$: $R^{n_i}\times\Sigma_i\times R\to R^{m_i\times m_i}$ and $h_i(\cdot)$: $R^n\times\Sigma_i\times R\to R^{m_i}$ such that

$$\Delta f_i(x_i,\sigma_i,t)=B_i(x_i,t)d_i(x_i,\sigma_i,t) \tag{3}$$

$$\Delta B_i(x_i,\sigma_i,t)=B_i(x_i,t)E_i(x_i,\sigma_i,t) \tag{4}$$

$$g_i(x,\sigma_i,t)=B_i(x_i,t)h_i(x,\sigma_i,t) \tag{5}$$

Remark 1. Eqs. (3)~(5) impose constraints on the structure of the uncertain part, Δf_i, ΔB_i, and interconnection, g_i once B_i is given. These conditions imply that the uncertainty and interconnection should lie within the range space of input matrix, B_i. In general, this property can be satisfied if B_i has high enough rank (i.e., the system has enough inputs).

3. Proposed Controllers

We first choose a input $\nu_i(\cdot)$: $R^{n_i}\times R\to R^{n_i}$ for the nominal system (i.e., the system in absence of uncertain part and interconnection) such that the controlled nominal system is asymptotically stable.

Assumption 2. There exist a C^1 functions $V_i(\cdot)$: $R^{n_i}\times R\to R_+$ and functions $\gamma_{1i}(\cdot)$, $\gamma_{2i}(\cdot)$, $\gamma_{3i}(\cdot)$: $R_+\to R_+$, where γ_{1i}, γ_{2i} belong to class KR (see Appendix) and γ_{3i} belongs to class K (see Appendix), such that for all $(\xi,t)\in R^{n_i}\times R$

$$\gamma_{1i}(\|\xi\|)\le V_i(\xi,t)\le\gamma_{2i}(\|\xi\|) \tag{6}$$

$$\frac{\partial V_i(\xi,t)}{\partial t}+\frac{\partial V_i(\xi,t)}{\partial\xi}\bar{f}_i(\xi,t)\le-\gamma_{3i}(\|\xi\|) \tag{7}$$

where

$$\bar{f}_i(\xi,t)=f_i(\xi,t)+B_i(\xi,t)\nu_i(\xi,t) \tag{8}$$

The following assumption is an additional condition imposed on the uncertainty and interconnection.

Assumption 3. There exists a known constant ρ_{E_i} such that for all $(x_i,\sigma_i,t)\in R^{n_i}\times\Sigma_i\times R$

$$\min_{\sigma_i\in\Sigma_i}\left\{\frac{1}{2}\lambda_m[E_i(x_i,\sigma_i,t)+E_i^T(x_i,\sigma_i,t)]\right\}\ge\rho_{E_i}>-1 \tag{9}$$

Assumption 4. There exist non-decreasing continuous functions $\psi_{ij}(\cdot)$: $R_+\to R_+$, $i,j\in N$ such that for all $(x,\sigma_i,t)\in R^n\times\Sigma_i\times R$

$$\max_{\sigma_i \in \Sigma_i} \|\Pi_i(x, \sigma_i, t)\| \leq \sum_{j=1}^{N} \psi_{ij}(\|x_j\|) \tag{10}$$

where

$$\Pi_i(x, \sigma_i, t) = d_i(x_i, \sigma_i, t) + E_i(x_i, \sigma_i, t) \cdot \nu_i(x_i, t) + h_i(x, \sigma_i, t) \tag{11}$$

The function $\psi_{ij}(\cdot)$in (10) can be determined from the boundary values of the uncertain parameters in of Π_i of(11)

Remark 2. The requirement (9) is to assure that a given control acts in the desired direction. In view of the condition (10), much more general class of uncertainty and interconnection is considered in this work. In Han and Chen (1991), the interconnection is bounded by a first order polynomial of $\|x_j\|$. In Chen *et al.* (1991), uncertainty and interconnection are both bounded by first order polynomials. In Shi and Singh (1991), the uncertain system has a linear nominal system and the interconnection is bounded by a p^{th} order polynomial. The current consideration renders all previous settings as special cases. Practical examples that fall into the current class of uncertain systems include, for example, robot hybrid control where uncertain portion is bounded by a second order polynomial (Chen and Pandey, 1990) and continuous stirred tank reactor (CSTR) control where interconnection is bounded by an exponential function (Shu et al., 1989).

Now a class of decentralized robust controllers are proposed as follows.

$$u_i(x_i, t) = \nu_i(x_i, t) + v_i(x_i, t) \tag{12}$$

$$v_i(x_i, t) = -\frac{1}{2} k_i (1 + \rho_{E_i})^{-1} \alpha_i(x_i, t) \beta_i(x_i, t) \tag{13}$$

$$\alpha_i(x_i, t) = B_i^T(x_i, t) \nabla_{x_i} V_i(x_i, t) \tag{14}$$

$$\beta_i(x_i, t) = \begin{cases} \eta_i(V_i) + \dfrac{N}{2k_i}[(\gamma_{3i} \circ \gamma_{2i}^{-1})(V_i)]^{-1} \sum_{j=1}^{N} [(\psi_{ji} \circ \gamma_{1i}^{-1})(V_i)]^2 \\ \quad \text{if } V_i > \gamma_{2i}(\varepsilon_i) \\ \eta_i(V_i) + \dfrac{N}{2k_i}[(\gamma_{3i}(\varepsilon_i)]^{-1} \sum_{j=1}^{N} [(\psi_{ji} \circ \gamma_{1i}^{-1})(V_i)]^2 \\ \quad \text{if } V_i \leq \gamma_{2i}(\varepsilon_i) \end{cases} \tag{15}$$

where the positive constants, k_i and ε_i, are chosen by a designer, the function $\eta_i(\cdot)$: $R_+ \to R_+$ is continuous and satisfies

$$r_1 \leq r_2 \Rightarrow \eta_i(r_1) \leq \eta_i(r_2),\ \forall r_1, r_2 \in R_+$$
$$\eta_i(0) > 0,\ r \geq 0 \Rightarrow \eta_i(r) > 0$$

In (15), the notation $f \circ g(x)$ denotes $f(g(x))$.

Remark 3. Notice that the state vector of the other subsystems, x_j, is not included in the proposed controller $u_i(\cdot)$ in (12). This implies that though one subsystem is interconnected with the other subsystems, only the information of its own state is utilized in its control design. In other words, there is no communication needed among the subsystems for the control to be implemented. The control $u_i(\cdot)$ is continuous and of saturation-type due to the function $\beta_i(\cdot)$ in (15).

4. Properties of Systems with Proposed Controllers

For convenience, we describe the large-scale system S in a compact form as

$$S:\ \dot{x}(t) = f(x, t) + \Delta f(x, \sigma, t) + [B(x, t) + \Delta B(x, \sigma, t)]u(t) + g(x, \sigma, t)$$
$$x(t_0) = x_0 \tag{16}$$

where

$$x_0 \equiv [x_{10}^T, x_{20}^T, \cdots, x_{N0}^T]^T \in R^n$$
$$u \equiv [u_1^T, u_2^T, \cdots, u_N^T]^T \in R^m,\ m = \sum_{i=1}^{N} m_i$$
$$\sigma \equiv [\sigma_1^T, \sigma_2^T, \cdots, \sigma_N^T]^T \in R^l,\ l = \sum_{i=1}^{N} l_i$$
$$f(x, t) \equiv [f_1(x_1, t)^T, f_2(x_2, t)^T, \cdots, f_N(x_N, t)^T]^T \in R^n$$
$$\Delta f(x, \sigma, t) \equiv [\Delta f_1(x_1, \sigma_1, t)^T, \Delta f_2(x_2, \sigma_2, t)^T, \cdots, \Delta f_N(x_N, \sigma_N, t)^T]^T \in R^n$$
$$B(x, t) \equiv \operatorname{diag}\{B_1(x_1, t), B_2(x_2, t), \cdots, B_N(x_N, t)\} \in R^{n \times m}$$
$$\Delta B(x, \sigma, t)^T \equiv \operatorname{diag}\{\Delta B_1(x_1, \sigma_1, t), \Delta B_2(x_2, \sigma_2, t), \cdots, \Delta B_N(x_N, \sigma_N, t)] \in R^{n \times m}$$
$$g(x, \sigma, t) \equiv [g_1(x, \sigma_1, t)^T, g_2(x, \sigma_2, t)^T, \cdots, g_N(x, \sigma_N, t)^T]^T \in R^n$$

The following definition describes the desired system behavior.

Definition 1. (Chen, 1986 and 1988; Corless and Leitmann, 1981 and 1988; Han, 1995) A feedback control

$$p(\cdot) = [p_1^T(\cdot), p_2^T(\cdot), \cdots, p_N^T(\cdot)]^T \tag{17}$$

$p_i(\cdot)$: $R^n \times R \to R^{m_i}$, $i = 1, 2, \cdots, N$, renders the uncertain system S

$$\dot{x}(t)=f(x, t)+\Delta f(x, \sigma, t)+[B(x, t)+\Delta B(x, \sigma, t)]p(x, t)+g(x, \sigma, t)$$
$$x(t_0)=x_0 \tag{18}$$

practically stable if and only if there exists an $r_0>0$ such that the following properties hold.

(i) Existence of solutions: The system (18) possesses a solution $x(\cdot): [t_0, \infty)\to R^n$.

(ii) Uniform boundedness: Given any $\underline{r}\in(0, \infty)$ and any solution $x(\cdot): [t_0, \infty)\to R^n$ of (18), there exists a $d(\underline{r})<\infty$ such that $\|x_0\|\le\underline{r}$ implies $\|x(t)\|\le d(\underline{r})$ for all $t\in[t, \infty)$.

(iii) Uniform ultimate boundedness: Given any and $\bar{r}>r_0$ and any $\underline{r}\in(0, \infty)$, there exists a finite time $T(\underline{r}, \bar{r})$ such that $\|x_0\|\le\underline{r}$ implies $\|x(t)\|\le\bar{r}$ for all $t\ge t_0+T(\underline{r}, \bar{r})$.

(iv) Uniform stabity: Given any $\bar{r}>r_0$, there exists a $\delta(\bar{r})>0$ such that $\|x_0\|\le\delta(\bar{r})$ implies $\|x(t)\|\le\bar{r}$ for all $t\ge t_0$.

Theorem 1. Subject to Assumption 1~4, the uncertain large-scale system (16), under the decentralized control (12)

$$u(x, t)=[u_1^T(x_1, t), u_2^T(x_2, t), \cdots, u_N^T(x_N, t)]^T$$

is practically stable.

Proof. See Appendix

Remark 4. The reason why the function $\beta_i(\cdot)$ in (15) is taken to be of saturation-type is that the function $[\gamma_{3i}(\cdot)]^{-1}\psi_{ji}^2(\cdot)$ is not assured to be well defined on $[0, \infty)$, especially at 0. The following assumption addresses a special class of the bounding functions under which non-saturation type robust control is also applicable.

Assumption 5. There exist a constant $\bar{\psi}_{i0}$ and non-decreasing continuous functions $\bar{\psi}_{ij}(\cdot)$: $R_+\to R_+$, $i, j\in N$, such that

$$\max_{\sigma_i\in\Sigma_i}\|\Pi(x, \sigma_i, t)\|\le\bar{\psi}_{i0}+\sum_{j=1}^{N}r_{3j}^{\frac{1}{2}}(\|x_j\|)\ \bar{\psi}_{ij}(\|x_j\|) \tag{19}$$

for all $(x, \sigma_i, t)\in R^n\times\Sigma_i\times R$.

Theorem 2. Subject to Assumption 1~3 and 5, under the decentralized control

$$u(x, t)=[u_1^T(x_1, t), u_2^T(x_2, t), \cdots, u_N^T(x_N, t)]^T$$
$$u_i(x_i, t)=\nu_i(x_i, t)+v_i(x_i, t)$$
$$v_i(x_i, t)=-\frac{1}{2}k_i(1+\rho_{Ei})^{-1}\alpha_i(x_i, t)\beta_i(x_i, t)$$
$$\beta_i(x_i, t)=\eta_i(V_i)+\frac{1}{2k_i}\sum_{j=1}^{N}\delta_j[(\bar{\psi}_{ji}\circ\gamma_{1i}^{-1})(V_i)]^2 \tag{20}$$

where $k_i>0$, $\delta_j\in(N, \infty)$, the uncertain large-scale system (16) is practically stable.

Proof. See Appendix

5. Specialization to Linear Systems

In this section, we cosider the special case that the nominal subsystems are linear and time-invariant.

$$S_i:\ \dot{x}_i=A_ix_i+\Delta f_i(x_i, \sigma_i, t)+[B_i+\Delta B_i(x_i, \sigma_i, t)]u_i+g_i(x, \sigma_i, t)$$
$$x_i(t_0)=x_{i0} \tag{21}$$

In Assumption 2, the nominal control is chosen as $\nu_i=J_ix_i$ such that $\bar{A}_i=A_i+B_iJ_i$ is Hurwitz and Lyapunov function is taken as $V_i(x_i)=x_i^TP_ix_i$ where $P_i>0$ is the unique solution of Lyapunov equation

$$P_i\bar{A}_i+\bar{A}_i^TP_i+Q_i=0,\ Q_i>0 \tag{22}$$

then the bounding functions $\gamma_{1i}(\cdot)$, $\gamma_{2i}(\cdot)$ and $\gamma_{3i}(\cdot)$ are of quadratic forms

$$\gamma_{1i}(\|x_i\|)=\lambda_m(P_i)\|x_i\|^2$$
$$\gamma_{2i}(\|x_i\|)=\lambda_M(P_i)\|x_i\|^2$$
$$\gamma_{3i}(\|x_i\|)=\lambda_m(Q_i)\|x_i\|^2 \tag{23}$$

We consider the same class of uncertainty and interconnection as that of Shi and Singh (1991).

Assumption 6. There exist positive constants ζ_{ijk}'s such that for all $(x, \sigma_i, t)\in R^n\times\Sigma_i\times R$

$$\max_{\sigma_i\in\Sigma_i}\|\Pi_i(x, \sigma_i, t)\|\le\sum_{j=1}^{N}\sum_{k=0}^{r}\zeta_{ijk}\|x_j\|^k \tag{24}$$

Then, two follwing controllers, saturation type and non-saturation type, are both applicable. First, the saturation type controller is

$$u_i(x_i, t)=J_ix_i-\frac{1}{2}k_i(1+\rho_{Ei})^{-1}\alpha_i(x_i, t)\beta_i(x_i, t) \tag{25}$$

$$\alpha_i(x_i, t)=B_i^T P_i x_i \tag{26}$$

$$\beta_i(x_i, t)=\begin{cases} \eta_i(V_i)+\dfrac{N\lambda_M(P_i)}{2k_i\lambda_m(Q_i)V_i}\displaystyle\sum_{j=1}^{N}\left[\sum_{k=0}^{r}\frac{\zeta_{jik}V_i^{\frac{k}{2}}}{\lambda_m(P_i)^{\frac{k}{2}}}\right]^2 \\ \text{if } V_i>\lambda_M(P_i)\varepsilon_i^2 \\ \eta_i(V_i)+\dfrac{N}{2k_i\lambda_m(Q_i)\varepsilon_i^2}\displaystyle\sum_{j=1}^{N}\left[\sum_{k=0}^{r}\frac{\zeta_{jik}V_i^{\frac{k}{2}}}{\lambda_m(P_i)^{\frac{k}{2}}}\right]^2 \\ \text{if } V_i\leq\lambda_M(P_i)\varepsilon_i^2 \end{cases} \tag{27}$$

The non-saturation type controller is

$$u_i(x_i, t)=J_i x_i-\frac{1}{2}k_i(1+\rho_{E_i})^{-1}\alpha_i(x_i, t)\beta_i(x_i, t) \tag{28}$$

$$\beta_i(x_i, t)=\eta_i(V_i)+\frac{1}{2k_i\lambda_m(Q_i)}\sum_{j=1}^{N}\delta_j\left[\sum_{k=1}^{r}\frac{\zeta_{jik}V_i^{\frac{k-1}{2}}}{\lambda_m(P_i)^{\frac{k-1}{2}}}\right]^2 \tag{29}$$

Here δ_j is a constant with $N<\delta_j<\infty$.

Remark 5. Shi and Singh (1991) also proposed a robust control for the systems considered above, whose nominal parts are linear and time-invariant and interconnections are bounded by the sum of polynomials of $\|x_j\|$. The proposed non-saturation type controller (28) is different from their one. In (28), $\alpha_i(x_i)$ is multiplied by $\beta_i(x_i)$ which is a $r-1$ order polynomial of $V_i^{0.5}(x_i)$, but in their control, $\alpha_i(x_i)$ is multiplied by $\|x_i\|^{r-1}$.

6. Illustrative Example

Consider the coupled inverted double pendulums which are subject to two distinct control inputs as shown in Fig. 1 (Gavel and Siljak, 1989; Chen et al., 1991). The pendulums are interconnected through the nonlinear spring whose reaction force is proportional to a square of the spring deflection. The position $a(t)\in[0, l]$ of the spring is unknown and time-varying. Furthermore, the payload m_i, $i=1,2$, is also unknown with $m_i=\bar{m}_i+\Delta m_i(t)$. Here $\bar{m}_i$ is the nominal payload and $\Delta m_i(t)$ is the uncertain payload. Using the state vector $x_i=(x_{i1}\ x_{i2})^T=(\theta_i\ \dot{\theta}_i)^T$, $i=1,2$, the equations of motion are described as

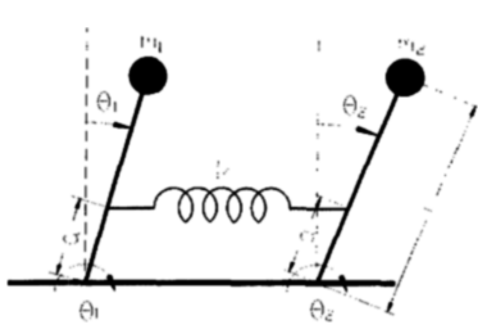

Fig. 1 Inverted double pendulums

$$S_1:\ \dot{x}_{11}=x_{12}$$

$$\dot{x}_{12}=\frac{g}{l}x_{11}+\frac{1}{m_1 l^2}u_1-\text{sign}(x_{11}-x_{21})\cdot\frac{ka^2}{m_1 l^2}(x_{11}-x_{21})^2$$

$$S_2:\ \dot{x}_{21}=x_{22}$$

$$\dot{x}_{22}=\frac{g}{l}x_{21}+\frac{1}{m_2 l^2}u_2-\text{sign}(x_{21}-x_{11})\cdot\frac{ka^2}{m_2 l^2}(x_{21}-x_{11})^2$$

The controls proposed by Gavel and Siljak (1989) and Chen *et al.* (1991) are not applicable since there is uncertainty $\Delta m_i(t)$ in the control channel and the interconnection is bounded by a second order polymonial.

For simulation purpose, we take $g=l=1$, $\bar{m}_1=1$ $\bar{m}_2=0.5$, $k=1$, $|\Delta m_1|\leq 0.1$, $|\Delta m_2|\leq 0.1$, and choose the uncertainties as $\Delta m_1=-0.1$, $\Delta m_2=0.1$, and $a(t)=0.5+0.5\sin(20t)$. The system is in the form of section 5 with

$$A_1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ B_1=\begin{bmatrix}0\\ 1\end{bmatrix},\ A_2=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ B_2=\begin{bmatrix}0\\ 2\end{bmatrix}$$

We choose

$$J_1=[-5\ \ -4],\qquad J_2=[-2.5\ \ -2]$$

$$\bar{A}_1=\begin{bmatrix}0 & 1\\ -4 & -4\end{bmatrix},\quad \bar{A}_2=\begin{bmatrix}0 & 1\\ -4 & -4\end{bmatrix}$$

such that all poles of the controlled nominal system locate at -2.

The solutions of the Lyapunov equations are given by ($Q_i=I_1$, $i=1,2$)

$$P_1=P_2=\begin{bmatrix}\frac{9}{8} & \frac{1}{8}\\ \frac{1}{8} & \frac{5}{32}\end{bmatrix}$$

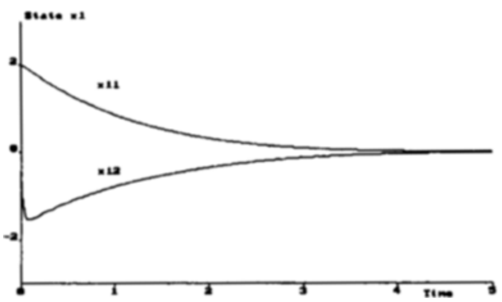

Fig. 2 State history of subsystem 1

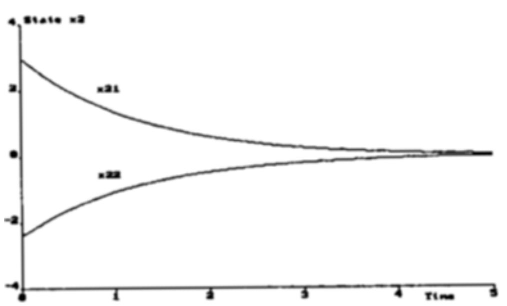

Fig. 3 State history of subsystem 2

The control is then obtained as ($k_i=1$, $\delta_i=3$, $i=1,2$)

$$u_1=-5x_{11}-4x_{12}-(0.069x_{11}+0.017x_{12})\cdot(1.76+12.46V_1^{0.5}+118.1V_1)$$

$$u_2=-2.5x_{21}-2x_{22}-(0.15x_{21}+0.038x_{22})\cdot(1.96+16.0V_2^{0.5}+118.2V_2)$$

where the Lyapunov functions are given by

$$V_1=1.125x_{11}^2+0.25x_{11}x_{12}+0.156\ x_{12}^2$$

$$V_2=1.125x_{21}^2+0.25x_{21}x_{22}+0.156\ x_{22}^2$$

Figs. 2 and 3 represent the controlled system responses x_1 and x_2, respectively.

6. Conclusions

The focus of the controllers presented in this work is on the compensation of uncertainty and strong interconnection among subsystems. We show specially how both internal and external uncertainties are taken into considerations. Only very mild condition is required for the interconnection bound, as the bound can be nonlinear. This work may cover a broad class of large-scale systems since the current consideration renders all previous settings as special cases. The design proposed here, as compared with other work in decentralized control, is especially useful as one intends to tackle strongly interconnected systems.

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Appendix

Definition 1. 1) A function $\gamma(\cdot): R_+ \to R_+$ belong to class K if and only if it is continuous and satisfies

$$r_1 \le r_2 \Rightarrow \gamma(r_1) \le \gamma(r_2) \forall r_1, r_2 \in R_+$$
$$\gamma(0)=0,\ r>0 \Rightarrow \gamma(r)>0$$

2) A function $\gamma(\cdot): R_+ \to R_+$ belongs to class KR if and only if it belong to K and $\lim_{r\to\infty} \gamma(r)=\infty$.

Proof of Theorem 1. Take the Lyapunov function candidate for the system (16)

$$V(x, t)=\sum_{i=1}^{N} k_i \phi_i(V_i) \tag{a.1}$$

where V_i is the Lyapunov function for the nominal system of ith subsystem and $\phi_i(\cdot)$ belongs to KR and strictly increasing since the derivative is taken as $\beta_i(x_i, t)$, that is,

$$\frac{d\phi_i(V_i)}{dV_i}=\begin{cases} \eta_i(V_i)+\frac{N}{2k_i}[(\gamma_{3i}\circ\gamma_{2i}^{-1})(V_i)]^{-1}\sum_{j=1}^{N}[(\psi_{ji}\circ\gamma_{1i}^{-1})(V_i)]^2 \\ \quad \text{if } V_i>\gamma_{2i}(\varepsilon) \\ \eta_i(V_i)+\frac{N}{2k_i}[(\gamma_{3i}(\varepsilon_i)]^{-1}\sum_{j=1}^{N}[(\psi_{ji}\circ\gamma_{1i}^{-1})(V_i)]^2 \\ \quad \text{if } V_i\le\gamma_{2i}(\varepsilon_i) \end{cases} \tag{a.2}$$

(notice that the derivative is positive.)

First, we consider the case that γ_{3i} belong to class KR, then extend it to the case that γ_{3i} belong to class K. For the former, $\eta_i(V_i)$ in (a.2) is chosen to be 1.

Subject to (6), it follows that

$$(\phi_i\circ\gamma_{1i})(\|x_i\|)\le\phi_i(V_i)\le(\phi_i\circ\gamma_{2i})(\|x_i\|) \tag{a.3}$$

Since the functions $(\phi_i\circ\gamma_{1i})(\cdot)$ and $(\phi_i\circ\gamma_{2i})(\cdot)$ are strictly increasing continuous, there exit $\gamma_1(\cdot)$: $R_+\to R_+$ and $\gamma_2(\cdot)$: $R_+\to R_+$ which belong to class KR such that

$$\begin{aligned}\gamma_1(\|x\|) &\le \sum_{i=1}^{N} k_i(\phi_i\circ\gamma_{1i})(\|x_i\|)\\ &\le \sum_{i=1}^{N} k_i\phi_i(V_i)\end{aligned} \tag{a.4}$$

$$\begin{aligned}\gamma_2(\|x\|) &\ge \sum_{i=1}^{N} k_i(\phi_i\circ\gamma_{2i})(\|x_i\|)\\ &\ge \sum_{i=1}^{N} k_i\phi_i(V_i)\end{aligned} \tag{a.5}$$

for any given uncertainty realization, the total time derivative of V along any trajectory of the controlled system S under (12) ~ (15) is given by

$$\begin{aligned}\dot V(x, t) &= \sum_{i=1}^{N} k_i\dot\phi_i(V_i)\\ &= \sum_{i=1}^{N} k_i\frac{d\phi_i}{dV_i}\dot V_i\\ &= \sum_{i=1}^{N} k_i\frac{d\phi_i}{dV_i}\Big[\frac{dV_i}{\partial t}+\frac{dV_i}{\partial x_i}(\bar f_i+B_i(I_i+E_i)\cdot\\ &\quad v_i+B_i\Pi_i)\Big]\\ &\le \sum_{i=1}^{N} k_i\frac{d\phi_i}{dV_i}\Big[-\gamma_{3i}-\frac{1}{2}k_i\frac{d\phi_i}{dV_i}\|\alpha_i\|^2\\ &\quad +\|\alpha_i\|\sum_{j=1}^{N}\psi_{ij}\Big]\end{aligned} \tag{a.6}$$

Using the inequality $2ab\le a^2+b^2$, a, $b,\in R$,

$$k_i\frac{d\phi_i}{dV_i}\|\alpha_i\|\sum_{j=1}^{N}\psi_{ij}=\sum_{i=1}^{N}k_i\frac{d\phi_i}{dV_i}\|\alpha_i\|N^{-\frac{1}{2}}N^{\frac{1}{2}}\psi_{ij}$$
$$\leq\sum_{j=1}^{N}\left[\frac{1}{2N}k_i^2\left[\frac{d\phi_i}{dV_i}\right]^2\|\alpha_i\|+\frac{N}{2}\psi_{ij}^2\right]$$
$$=\frac{1}{2}k_i^2\left[\frac{d\phi_i}{dV_i}\right]^2\|\alpha_i\|^2+\frac{N}{2}\sum_{j=1}^{N}\psi_{ij}^2 \quad (a.7)$$

Upon introducing (a.7), one has

$$\dot V(x,\ t)\leq-\sum_{i=1}^{N}k_i\frac{d\phi_i}{dV_i}\gamma_{3i}+\frac{N}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\psi_{ji}^2$$
$$=\sum_{i=1}^{N}\left[-k_i\frac{d\phi_i}{dV_i}\gamma_{3i}+\frac{N}{2}\sum_{j=1}^{N}\psi_{ji}^2\right] \quad (a.8)$$

If $V_i(x_i,\ t)>\gamma_{2i}(\varepsilon_i)$, by (a.2),

$$-k_i\frac{d\phi_i}{dV_i}\gamma_{3i}+\frac{N}{2}\sum_{j=1}^{N}\psi_{ji}^2$$
$$=-k_i\gamma_{3i}-\frac{N}{2}\gamma_{3i}[(\gamma_{3i}\circ\gamma_{2i}^{-1})(V_i)]^{-1}\cdot$$
$$\sum_{j=1}^{N}[(\psi_{ji}\circ\gamma_{1i}^{-1})(V_i)]^2+\frac{N}{2}\sum_{j=1}^{N}\psi_{ji}^2 \quad (a.9)$$

From (6) in Assumption 2, it follows that

$$\gamma_{2i}^{-1}(V_i)\leq\|x_i\|\leq\gamma_{1i}^{-1}(V_i) \quad (a.10)$$

By (a.10),

$$\gamma_{3i}(\|x_i\|)[(\gamma_{3i}\circ\gamma_{2i}^{-1})]^{-1}\geq 1 \quad (a.11)$$
$$(\psi_{ji}\circ\gamma_{1i}^{-1})(V_i)\geq\psi_{ji}(\|x_i\|) \quad (a.12)$$

Note that this holds for both $V_i(x_i,\ t)>\gamma_{2i}(\varepsilon_i)$ and $V_i(x_i,\ t)\leq\gamma_{2i}(\varepsilon_i)$. It follows that

$$-\frac{N}{2}[(\gamma_{3i}\circ\gamma_{2i}^{-1})(V_i)]^{-1}\sum_{j=1}^{N}[(\psi_{ji}\circ\gamma_{1i}^{-1})(V_i)]^2$$
$$+\frac{N}{2}\sum_{j=1}^{N}\psi_{ji}^2\leq 0 \quad (a.13)$$

Consequently, if $V_i(x_i,\ t)>\gamma_{2i}(\varepsilon_i)$,

$$-k_i\frac{d\phi_i}{dV_i}\gamma_{3i}+\frac{N}{2}\sum_{j=1}^{N}\psi_{ji}^2\leq-k_i\gamma_{3i} \quad (a.14)$$

If $V_i(x_i,\ t)\leq\gamma_{2i}(\varepsilon_i)$,

$$-k_i\frac{d\phi_i}{dV_i}\gamma_{3i}+\frac{N}{2}\sum_{j=1}^{N}\psi_{ji}^2$$
$$=-k_i\gamma_{3i}-\frac{N}{2}\gamma_{3i}[(\gamma_{3i}(\varepsilon_i)]^{-1}\sum_{j=1}^{N}[(\psi_{ji}\circ\gamma_{1i}^{-1})\cdot$$
$$(V_i)]^2+\frac{N}{2}\sum_{j=1}^{N}\psi_{ji}^2 \quad (a.15)$$

By (a.10), it follows that

$$(\psi_{ji}\circ\gamma_{1i}^{-1})(V_i)\geq\psi_{ji}(\|x_i\|) \quad (a.16)$$
$$\psi_{ji}(\|x_i\|)\leq(\psi_{ji}\circ\gamma_{1i}^{-1}\circ\gamma_{2i})(\varepsilon_i) \quad (a.17)$$

Upon introducing (a.16) and (a.17), if $V_i(x_i,\ t)\leq\gamma_{2i}(\varepsilon_i)$,

$$-k_i\frac{d\phi_i}{dV_i}\gamma_{3i}+\frac{N}{2}\sum_{j=1}^{N}\psi_{ji}^2$$
$$\leq-k_i\gamma_{3i}-\frac{N}{2\gamma_{3i}(\varepsilon_i)}\sum_{j=1}^{N}\gamma_{3i}\psi_{ji}^2(\|x_i\|)+c_i \quad (a.18)$$

where

$$c_i=\frac{N}{2}\sum_{j=1}^{N}[(\psi_{ji}\circ\gamma_{1i}^{-1}\circ\gamma_{2i})(\varepsilon_i)]^2 \quad (a.19)$$

For any $x\in R^n$, without losing generality, we can consider that

$$V_i(x_i,\ t)>\gamma_{2i}(\varepsilon_i),\ i=1,\ 2,\ \cdots,q \quad (a.20)$$
$$V_i(x_i,\ t)\leq\gamma_{2i}(\varepsilon_i),\ i=q+1,\ q+2,\ \cdots,N \quad (a.21)$$

Then, by (a.14) and (a.18),

$$\dot V(x,\ t)\leq-\left\{\sum_{i=1}^{N}k_i\gamma_{3i}+\frac{N}{2}\sum_{i=q+1}^{N}\frac{1}{\gamma_{3i}(\varepsilon_i)}\cdot\left[\sum_{j=1}^{N}\gamma_{3i}\psi_{ji}^2\right]\right\}+\sum_{i=q+1}^{N}c_i \quad (a.22)$$

Since all functions on the right-hand-side of (a. 23) (except the last constant term) are continuous and strictly increasing, it follows that there exists a strictly increasing continuous function $\gamma_3(\cdot)$: $R_+\to R_+$ such that

$$\gamma_3(\|x\|)\leq\sum_{i=1}^{N}k_i\gamma_{3i}+\frac{N}{2}\sum_{i=q+1}^{N}\frac{1}{\gamma_{3i}(\varepsilon_i)}\left[\sum_{j=1}^{N}\gamma_{3i}\psi_{ji}^2\right] \quad (a.23)$$

Finally, it turns out that for all $(x,\ t)\in R^n\times R$

$$\dot V(x,\ t)\leq-\gamma_3(\|x\|)+C \quad (a.24)$$

where

$$C=\sum_{i=q+1}^{N}c_i \quad (a.25)$$

In other words, $\dot V$ is negative definite as $\|x\|$ is sufficiently large.

If γ_{3i} belongs to class K, it is not guaranteed that in $\gamma_3(\|x\|)$ in (a.23) goes to infinity as $\|x\|$ goes to infinity. Then, it may happen that the limity of $\gamma_3(\|x\|)$ is less than C. In this case, if we choose $\eta_i(\cdot)$ in (a.2) to be a function that belongs to class KR, this problem can be cleared.

In view of Corless and Leitmann (1981) and Chen (1988), By (a.4), (a.5), and (a.24), Theorem 1 has been proved.

Proof of Theorem 2. This analysis is similar with that of Theorem 1. In view of (a.6),

$$\dot{V}(x,t) \le \sum_{i=1}^{N} k_i \frac{d\phi_i}{dV_i}\Big[-\gamma_{3i} - \frac{1}{2}k_i \frac{d\phi_i}{dV_i}\|\alpha_i\|^2 + \bar{\psi}_{i0}\|\alpha_i\| + \|\alpha_i\| \sum_{j=1}^{N}\gamma_{3j}^{\frac{1}{2}}\bar{\psi}_{ij}\Big] \quad (a.26)$$

Using the inequality $2ab \le a^2 + b^2$, $a, b, \in R$,

$$k_i \frac{d\phi_i}{dV_i}\|\alpha_i\|\sum_{j=1}^{N}\gamma_{3j}^{\frac{1}{2}}\bar{\psi}_{ij} = \sum_{j=1}^{N} k_i \frac{d\phi_i}{dV_i}\|\alpha_i\|\delta_i^{-\frac{1}{2}}\delta_i^{\frac{1}{2}}\gamma_{3i}^{\frac{1}{2}}\bar{\psi}_{ij}$$
$$\le \sum_{j=1}^{N}\Big[\frac{1}{2}k_i^2\Big(\frac{d\phi_i}{dV_i}\Big)^2\|\alpha_i\|^2\frac{1}{\delta_i} + \frac{\delta_i}{2}\gamma_{3j}\bar{\psi}_{ij}^2\Big]$$
$$= \frac{N}{2\delta_i}k_i^2\Big(\frac{d\phi_i}{dV_i}\Big)^2\|\alpha_i\|^2 + \frac{\delta_i}{2}\sum_{j=1}^{N}\gamma_{3j}\bar{\psi}_{ij}^2 \quad (a.27)$$

Upon using (a.27) in (a.26),

$$\dot{V}(x,t) \le -\sum_{i=1}^{N}k_i\frac{d\phi_i}{dV_i}\gamma_{3i} + \sum_{i=1}^{N}\frac{\delta_i}{2}\sum_{j=1}^{N}\gamma_{3j}\bar{\psi}_{ij}^2 + \sum_{i=1}^{N}\Big[-\frac{1}{2}\Big(1-\frac{N}{\delta_i}\Big)k_i^2\Big(\frac{d\phi_i}{dV_i}\Big)^2\|\alpha_i\|^2 + \bar{\psi}_{i0}k_i\frac{d\phi_i}{dV_i}\|\alpha_i\|\Big] \quad (a.28)$$

By (20) ($\eta_i(V_i)$ is taken to be 1),

$$-\sum_{i=1}^{N}k_i\frac{d\phi_i}{dV_i}\gamma_{3i} + \sum_{i=1}^{N}\frac{\delta_i}{2}\sum_{j=1}^{N}\gamma_{3j}\bar{\psi}_{ij}^2$$
$$= \sum_{i=1}^{N}\Big[-k_i\frac{d\phi_i}{dV_i}\gamma_{3i} + \gamma_{3i}\sum_{j=1}^{N}\frac{\delta_j}{2}\bar{\psi}_{ji}^2\Big]$$
$$= -\sum_{i=1}^{N}k_i\gamma_{3i} - \sum_{i=1}^{N}\gamma_{3i}\Big[\sum_{j=1}^{N}\frac{\delta_i}{2}([(\bar{\psi}_{ji}\circ\gamma_{1i}^{-1})(V_i)]^2 - \bar{\psi}_{ji}^2)\Big] \le -\sum_{i=1}^{N}k_i\gamma_{3i} \quad (a.29)$$

The inequality in (a.29) is due to the fact that

$$[(\bar{\psi}_{ji}\circ\gamma_{1i}^{-1})(V_i)]^2 - \bar{\psi}_{ji}^2(\|x_i\|) \ge 0 \quad \forall x_i,\ t \quad (a.30)$$

The argument is similar to (a.10) ~ (a.12). The second term of the right-hand side of (a.28) is bounded by a constant as shown in the following:

$$-\frac{1}{2}\Big(1-\frac{N}{\delta_i}\Big)k_i^2\Big(\frac{d\phi_i}{dV_i}\Big)^2\|\alpha_i\|^2 + \bar{\psi}_{i0}k_i\frac{d\phi_i}{dV_i}\|\alpha_i\| \le \frac{\bar{\psi}_{i0}^2}{2\Big(1-\frac{N}{\delta_i}\Big)} \quad (a.31)$$

Then, by combining (a.28), (a.29), and (a.31), one has

$$\dot{V}(x,t) \le -\sum_{i=1}^{N}k_i\gamma_{3i}(\|x_i\|) + \bar{C} \le -\gamma_3(\|x\|) + \bar{C} \quad (a.32)$$

where

$$\bar{C} = \frac{\bar{\psi}_{i0}^{\,2}}{2\Big(1-\frac{N}{\delta_i}\Big)} \quad (a.33)$$

and the strictly increasing continuous function $\gamma_3(\cdot)$ which belongs to class KR can be chosen such that

$$\gamma_3(\|x\|) \le \sum_{i=1}^{N}k_i\gamma_{3i}(\|x_i\|) \quad \forall x \quad (a.34)$$

Consequently, the result of Theorem 2 follows (see the proof of Theorem 1).